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TURBULENT HEAT-TRANSFER CHARACTERISTICS OF VISCOELASTIC FLUIDS

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Abstract—An analysis of experimental heat-transfer results taken over fairly broad ranges of the pertinent variables is presented for purposes of establishing the major heat-transfer characteristics of these systems and delineating the areas in which further and more detailed studies may be helpful. These results, when analyzed in terms of the analogy equations, suggest major decreases to occur in the eddy transport coefficients, at least in wall regions, when the fluid elasticity level, as measured by its relaxation time, is increased to the level of 10^{-3}–10^{-2} s. This tentative conclusion is of importance from the viewpoint of defining in part the structure of the turbulence in these systems, as direct measurements with the usual kinds of turbulence probes may be difficult or even impossible to carry out. Thus the present results also suggest that the very low drag coefficients frequently observed with these systems do not arise as a result of a "conservative", as opposed to a "dissipative", turbulent field but rather because of the strong suppression of turbulence in the fluid when the Deborah number of the flow becomes sufficiently great.

NOMENCLATURE

C_p, specific heat;
D, internal tube diameter;
E, eddy diffusivity, assumed equal for momentum and heat transfer;
f, fanning friction factor, $f = \tau_w/(\rho V^2/2g_c)$;
g_c, dimensional conversion factor;
G, mass velocity of fluid;
h_x, local heat-transfer coefficient;
h_s, steady-state or well-developed heat-transfer coefficient;
k, thermal conductivity of fluid;
K, K', consistency indices, equations (1) and (2);
n, n', flow behavior indices, equations (1) and (2);
N, frequency of turbulent velocity fluctuations;
Nu_s, steady-state or well-developed Nusselt number, h_sD/K;
Nu_x, local Nusselt number;
Pr, Newtonian-Prandtl number evaluated at mean (bulk) fluid temperature. Non-Newtonian-Prandtl number evaluated at mean fluid temperature and local value of shearing stress;
Pr_w, Prandtl number evaluated at τ_w (wall shearing stress) and at mean fluid temperature;
q, heat flux based on temperature rise of fluid;
q_e, heat flux based on electrical input;
q_{mo}, local transport rate by molecular conduction;
q_t, local transport rate by turbulent processes;
Re, Reynolds number, evaluated at bulk mean temperature;
Re', generalized Reynolds number,

$$Re' = \frac{D^{n'}V^{2-n'}\rho}{g_c K' 8^{n'-1}};$$

St, Stanton number h_s/C_pG;
T_c, centerline temperature;
T_m, mean fluid temperature;
T_{wi}, local inner wall temperature;
u, local velocity;

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u^*, friction velocity $\sqrt{(g_c \tau_w/\rho)}$;
u^+, u/u^*;
U, maximum (centerline) velocity;
V, mean velocity;
x, axial position.

Greek symbols
$\dot{\gamma}$, shear rate;
$\dot{\gamma}_w$, shear rate evaluated at the wall stress level;
θ_{fl}, relaxation time of fluid;
θ_m, $(T_m - T_{w_i})/(T_c - T_{w_i})$;
μ, Newtonian viscosity evaluated at bulk temperature;
μ_w, Newtonian viscosity evaluated at wall temperature;
μ_{aw}, apparent viscosity evaluated at mean temperature and at wall stress level;
τ, shear stress;
τ_w, shear stress evaluated at wall;
ρ, density of fluid;
$1/\phi_m$, ratio of maximum-to-mean velocity, U/V.

INTRODUCTION

A NUMBER of analyses supported by experimental results are available in the area of heat transfer to purely viscous fluids under both laminar and turbulent flow conditions. Of these, the ones dealing with non-Newtonian fluids have recently been compiled and reviewed elsewhere [21, 27, 28]. However no published experimental results of any kind were available for the interesting case of viscoelastic fluids when this work was undertaken and very recent activities [1, 4, 22, 42] serve to resolve this need only in part. In view of the interesting phenomenon of turbulent drag reduction (see, for example [5, 6, 30, 32–34]) exhibited by viscoelastic fluids, heat-transfer studies under turbulent flow conditions may be of interest from the viewpoint of their pragmatic value and may in addition serve as a probe with which to study the turbulent behavior of these systems.

To interpret heat-transfer measurements a knowledge of the physical properties of the viscoelastic fluids used is necessary. Significant studies have recently been reported; rather complete measurements of the fluid properties under steady laminar shearing flow conditions have been carried out on concentrated [13, 19, 33, 35, 36, 38] as well as very dilute [27] polymeric solutions. If the rheological properties of the fluid are expressible in terms of very simple constitutive equations then the parameters as determined in these studies may also suffice to portray the fluid properties under all other flow conditions. Whether this is the case or not for the real fluids of interest largely remains to be determined; the available studies under well-defined unsteady flow conditions [11, 18, 38, 41], though moderately extensive, do not appear to have defined any major effects not expected on the basis of steady laminar flow measurements alone. On the other hand, in turbulent fields major effects have been noted which may not have been predicted on the basis of laminar physical property measurements [30], and their origin evidently remains to be determined, suggesting the need for work outside the ranges of conditions employed to date by rheologists. Thus from three pertinent viewpoints: pragmatic importance of the problem, possible insight into the characteristics of the fluid used and the importance of heat-transfer measurements as a "probe" for understanding the structure of the turbulent fluid fields, the present time seems a propitious one for studies of the heat-transfer characteristics of turbulent viscoelastic fluids. This paper is an initial step in such a direction, aimed primarily at scouting the field as a whole in order to define major effects and the conditions under which these appear.

EXPERIMENTAL EQUIPMENT AND PROCEDURES

A horizontal stainless steel tube (heated electrically by using the tube wall as the resistor) provided the test-section with which measurements were made. This test section was fitted into a recirculating flow loop comprising a mixing and storage tank, a pump, a flowmeter, and a heat exchanger for returning the fluid to its initial conditions.

A 76-in long calming section of stainless steel $\frac{7}{8}$-in tubing (I.D. 0·745 in, O.D. 0·875 in) followed the discharge line from the pump. Seven pressure tap holes of $\frac{1}{4}$-in dia. were drilled at 12-in intervals; these holes were intentionally made of large diameter to permit rapid equilibration even with viscous fluids. As will be noted later holes of this size do not appear to affect either the readings taken or the flow field in any adverse manner. Burrs on the inside of the tube due to drilling were carefully removed. Stainless steel pipe stubs ($\frac{1}{8}$-in dia.) 6-in long were then silver soldered at each hole. These pressure taps were used to obtain the friction factor at each Reynolds number independently of any empirical correlations and to check whether the velocity distribution was well-developed before the fluid entered the heated test section (i.e. whether the friction factor had leveled out to a constant value). A brass flange was silver-soldered to the downstream end of the calming section to secure it to the test section, the latter consisting of a 36-in long piece of $\frac{7}{8}$-in stainless steel tubing (type 304). Iron–constantan thermocouples of 30 gage wire were prepared by electrically fusing the joints. Thirty-four of these thermocouples were then spot-welded on the outer wall of the test section at seventeen axial locations (two thermocouples were located diametrically opposite each other at each axial location). Extreme care was taken to keep the level of heat as low as possible to avoid any distortion of the tube during spot-welding. After achieving spot-welded joints, each thermocouple was wrapped twice around the tube and then led off to a thermocouple switch. The test section electrical end connections consisted of $12 \times 12 \times \frac{1}{8}$-in copper terminal plates. To supply electrical power to the test section at a very low voltage (about 5–10 V) and high current (up to 2500 A), $4 \times \frac{3}{8}$-in copper sections were used as bus bars. Each copper terminal plate was attached to the bus bar by means of copper brackets at eight locations around the plate.

Standard fiberglass pipe insulation was used to insulate the test section and the calming section thermally. To insulate the test section unit thermally and electrically from the rest of the flow loop, $\frac{3}{4}$-in Teflon inserts were used on both ends of the test section. The inlet temperature of the fluid entering the test section was measured by means of a thermocouple positioned in a well situated at the discharge side of the pump. The line leading from the discharge of the pump to the test section was well insulated. The outlet temperature was measured by a thermocouple placed in a well downstream of a chamber immediately following the test section.

A Shirley–Ferranti viscometer was used to obtain the viscometric properties of the three polymeric solutions used in this study. These fluids were solutions of ET597, a water soluble partially hydrolyzed polyacrylamide of high molecular weight provided by the Dow Chemical Company. Solutions of this polymer are highly viscoelastic and evidently much more strongly resistant to mechanical and biological degradation than are most other similarly elastic polymers. Furthermore, the elastic properties have been measured over the concentration range from 0·01 to 0·80 per cent [26, 33, 35, 39] thus providing fluids which, comparatively speaking, have been far better characterized under conditions of large deformation and high deformation rates than any other available viscoelastic material.

All viscometric and heat transfer measurements on the viscoelastic fluids were preceded by corresponding measurements using water to validate all experimental procedures and equipment. Checks were made of the viscometric properties before and after running the heat-transfer loop in order to evaluate the fluid stability. All test runs on a given fluid were completed in less than a week; over this period no changes in the viscometric properties occurred. As noted elsewhere [39] such viscometric measurements are not a very sensitive test of fluid degradation but the absence of systematic changes in the turbulent pressure drops, with age of the fluid, tends to support this conclusion in the present case and possibly provides a more sensitive test of fluid stability.

Full details concerning the experimental equipment, procedures and results are available [15].

TREATMENT OF DATA

A total of 41 runs were made; the principal characteristics of these are summarized in Table 1. Though the energy balances were off by as much as 15 and 17 per cent in two cases more generally the heat transferred as computed by the electrical energy input agreed very closely with that computed from the temperature rise of the fluid. These balances also verify the assumed value of unity for the specific heat of the fluids used, in agreement with the measurements of Vaughn [40], Metzner and Friend [23] and Gluck [14].

Correspondingly, the other fluid physical properties must be known. Thermal conductivities of dilute polymeric solutions have been measured by a number of other investigators [7, 8, 14, 23, 27, 40] and were generally found to be indistinguishable from those of the solvent at the same temperature level and, in fact, are not considered further in the most recent publication in this area [28]. Densities were taken as identical to those of water.

The viscometric fluid properties (i.e. the shear stress–shear rate curves) are shown as a function of temperature in Fig. 1 for the fluid of principal interest, the 0·05% solution. Similar measurements were made for the other solutions [15]. There is some scatter of data points at very low shear rates because the measuring unit in the Shirley–Ferranti viscometer was not steady under these conditions. All the data points were approximated by means of straight lines and when the ranges of shear stress obtainable from the viscometer did not cover the actual working range of shear stress in the heat-transfer apparatus, the straight lines were extended to obtain the necessary results. Such extrapolations were primarily needed for 0·01% ET597 and, in this case, it is expected to be sufficiently accurate because the fluid properties of 0·01% ET597 are not very different from those of water. In the extreme case, the extrapolation was carried to a shear rate of 94000 s^{-1} whereas the experimental data were available only to 17500 s^{-1}. For the 0·45% ET597, the average inner wall temperature for some runs was higher than the range of temperatures studied in the cone and plate viscometer. Curves of shear stress vs. the reciprocal of the absolute temperature, with shear rate as parameter, were used to determine, by extrapolation, the physical properties at the higher temperatures. Although these extrapolations were not completely satisfactory, the actual measurement of viscometric data at these higher temperatures was not possible on the cone and plate viscometer because of evaporation of the fluid sample. In the worst case, the extrapolation had to be carried to 0·00155°R^{-1} whereas the experimental data were available to 0·0017°R^{-1}, the change in shear stress over this temperature

Table 1. Experimental conditions

Fluid used	Number of runs	Energy input *kw*†	Reynolds number (Re') range	Energy balance check (q_e/q)ave.	Maximum error
Water	6	9	19200–139000	1·09	17 per cent
0·01% ET597	11	9	18500 – 91800	0·99	8 per cent
0·05% ET597	11	6	17500–100500	1·02	8 per cent
0·45% ET597	13	4	774 – 10300	1·03	15 per cent

† Approximate only—varied slightly from run to run.

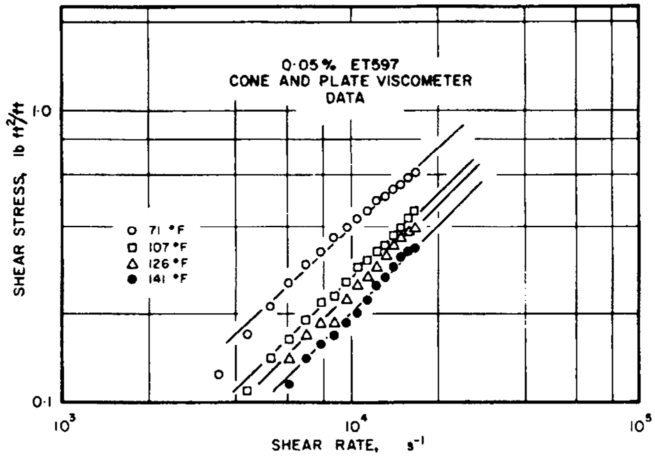

FIG. 1. Viscometric properties of 0·05% polymeric solution.

range being 37 per cent. In view of the linearity of the curves the maximum error likely to be incurred as a result of such extrapolation was not more than 10 per cent. In most cases, the extrapolation was smaller.

The power law model for a non-Newtonian fluid may be written:

$$\tau = K(\dot{\gamma})^n \tag{1}$$

in which $\dot{\gamma}$ denotes the shear rate, K the consistency index, and n the flow behavior index. It is, however, more convenient to use K' and n' as defined by the following equation [9]:

$$\tau_w = K'(8V/D)^{n'} \tag{2}$$

in which V and D refer to the mean fluid velocity and the tube diameter, respectively. For the power law model:

$$n = n' \tag{3}$$

and

$$K' = K\left(\frac{3n' + 1}{4n'}\right)^{n'} \tag{4}$$

since

$$\gamma_w = \left(\frac{3n' + 1}{4n'}\right)\frac{8V}{D}. \tag{5}$$

In evaluating the generalized Reynolds number the fluid properties at the bulk temperature were used; Prandtl numbers were evaluated using the apparent viscosity taken at the wall shearing stress in all cases, since, for high Prandtl number systems the controlling resistance to heat transfer occurs near the wall. This comparative neglect of fluid properties at other radial positions has been considered carefully recently [28] and cannot be justified in all instances but is quite adequate for the purposes of the present study as the dilute solutions of primary primary interest have nearly constant viscosities. Either wall, film or bulk temperatures were used in evaluating properties as specified by the particular equation used. Inside wall temperatures were evaluated from the measured outer wall thermocouple readings using the standard analysis for conduction through a cylindrical wall serving as a heat source [20].

Constancy of the heat flux along the tube wall was verified by measuring the voltage distribution using the thermocouples as electrical leads. Local heat-transfer coefficients were obtained by dividing this flux by the local radial temperature difference, $T_{wi} - T_m$.

RESULTS, DISCUSSION AND ANALYSIS

Evaluation of techniques and equipment

The six runs made for this purpose, using water as a test fluid, are depicted in Fig. 2. They are compared with predictions based upon three equations:

Reichardt (Friend) equation:

$$Nu_s = \frac{(f/2)(\mu/\mu_w)^{0.14}(C_p\rho VD/k)}{1.18 + 11.8(\sqrt{f/2})(Pr - 1)(Pr)^{-\frac{1}{3}}}, \quad (6)$$

Dittus–Boelter equation:

$$Nu_s = 0.023\, Re^{0.8}\, Pr^{0.4}, \quad (7)$$

Sieder–Tate equation:

$$Nu_s = 0.027\, Re^{0.8}\, Pr^{\frac{1}{3}}\, (\mu/\mu_w)^{0.14}. \quad (8)$$

The maximum deviation between any predicted Nusselt number and the measured results

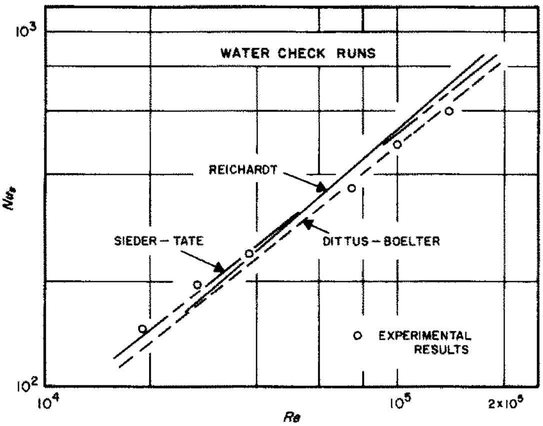

FIG. 2. Comparison of experimental water results with established predictions.

was 15·5 per cent. In general, the data are high at the lowest Reynolds numbers, possibly indicative of the presence of natural convection effects not accounted for in any of the above equations. If present, such effects would be revealed in asymmetric temperature profiles [20] and the double thermocouple readings at each axial position along the tube wall support this assumption. Thus, in general, the agreement shown in Fig. 2 is concluded to be a satisfactory index of valid performance.

Results for viscoelastic fluids

Figure 3 shows a typical axial temperature profile and Fig. 4 the variation of the local Nusselt number with axial position as derived from the data of Fig. 3. One notes the results to be well-defined; this was true in all instances though with the 0·45% polymeric solution fully developed (constant) Nusselt numbers were not attained in the test section length used.

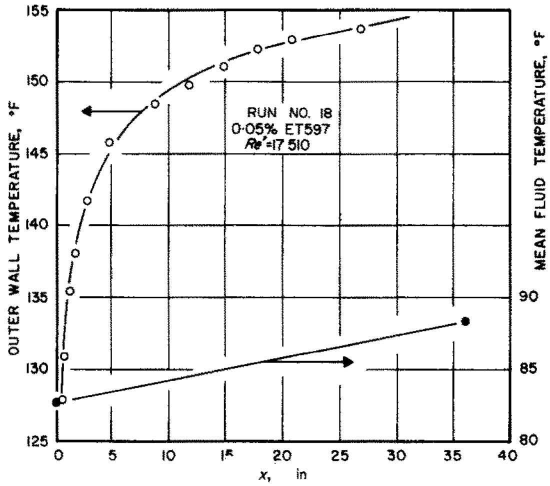

FIG. 3. Typical axial temperature profiles.

Pressure drop and heat-transfer coefficients are summarized on Figs. 5 and 6; dimensionless drag coefficients are given on Fig. 7. Figures 5 and 7 depict clearly the turbulent drag reduction characteristics of the fluids used and illustrate the reasons for the fluid concentrations chosen: the 0·01% solution shows only a very slight drag reduction at the highest flowrates used; at the other extreme the 0·45% solution, though showing a transition from laminar flow at a Reynolds number level of about 4000 exhibits friction coefficients very close to those of a completely laminar flow. These two fluids thus appear to bracket the entire range of the possible turbulent characteristics of these materials and the 0·05% solution represents an intermediate situation.

It is clear from examination of Figs. 5 and 6 that at a given flowrate the reduction in heat-transfer rates is far greater than the reduction in the turbulent drag or pressure drop; Table 2 summarizes such results quantitatively. It thus appears that these particular systems are disadvantageous where it is desired to obtain a maximum heat transfer for minimum pumping power. Correspondingly, if the heat-transfer and drag coefficient results are combined on conventional j-factor–Reynolds number curves the j-factors for heat transfer fall well below the corresponding $f/2$ values for the same fluid for both the 0·05 and 0·45% polymeric solutions. Although no formulation of the actual heat-transfer rate magnitudes which may be of predictive value has resulted from either this study or the concomitant work of Astarita and Marrucci [4] presumably empirical formulations would contain, in addition to the dimensionless groups appearing in equation [6], a Deborah number to portray the relative importance of the elastic and the viscous fluid properties [3, 5, 24, 25] and possibly additional groups representing dimensionless ratios of material property parameters [24]. Insufficient data are in hand to prepare such correlations, and theoretical analyses await a better knowledge of the structure of the turbulent velocity fields. Some initial steps may however be taken in this direction, employing the results depicted in Figs. 5–7, as follows:

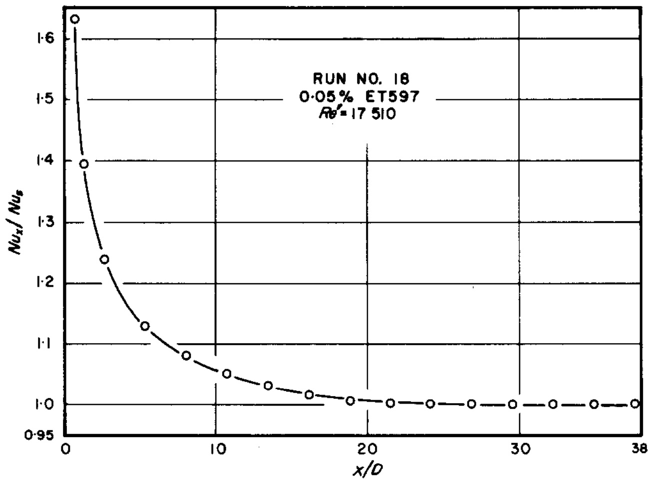

FIG. 4. Typical variation of local heat-transfer coefficient with axial position along tube.

The Reichardt equation in the form given in equation [6] has been previously used to interpret high Prandtl number heat-transfer measurements on purely viscous fluids, both Newtonian [12] and non-Newtonian [23], and, in a slightly modified form, by Peterson and Christiansen [28]. Under the conditions of the present experiments the modified form differs only insignificantly from equation (6). In the case of the work on non-Newtonian systems, the mean deviation of the measured heat-transfer rates from the predicted values was found to be 15–18 per cent; while this is not particularly good no improved analysis, theoretical or empirical, has been presented for turbulent non-Newtonian heat-transfer predictions, and much of this variation appears to be due to the fairly large random scatter in the data. As a result equation (6) will be used as the only available primary standard with which to evaluate the results for the present viscoelastic systems.

This equation may be written in more detail as:

$$St = \frac{(f/2)(1/\theta_m)}{(1/\phi_m) + (\sqrt{f/2})(Pr_w - 1)I} \quad (6a)$$

in which $1/\phi_m$ denotes the ratio of the maximum to the mean velocity, U/V, for the turbulent velocity field; θ_m the corresponding mean to

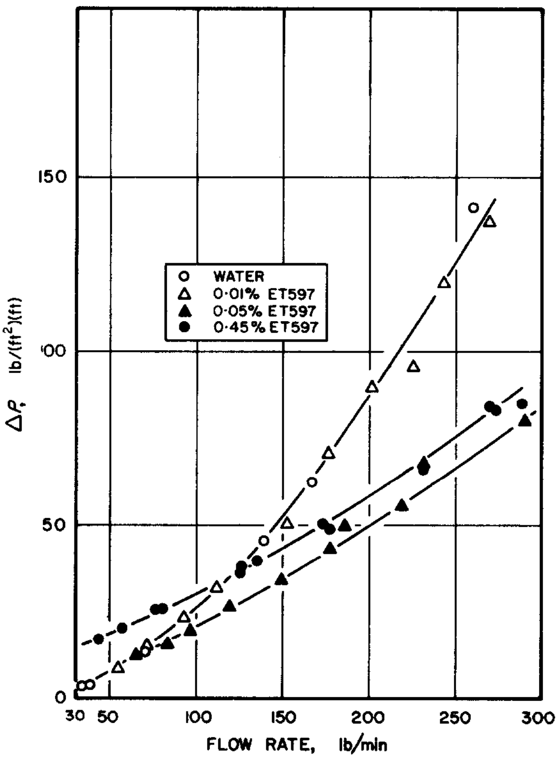

FIG. 5. Measured pressure drop—flowrate relationships as a function of concentration.

maximum temperature difference (radially) at any given axial position and I the integral:

$$I = \int_0^{U/u^*} \frac{(Pr - 1)\,du^+}{(Pr_w - 1)(Pr_w\,\rho E/\mu_{aw} + 1)}. \quad (7)$$

The molecular Prandtl number $C_p\mu_a/k$ is a function of radial position as a result of the dependence of the non-Newtonian viscosity μ_a upon shearing stress, hence radial position. For the two more dilute solutions this variation is only minor; for the 0·45% solution well-developed temperature profiles were not attained and equations (6a) and (7) are not relevant. Thus, as a good approximation for the dilute solutions one may write:

$$I = \int_0^{U/u^*} \frac{du^+}{Pr_w\,\rho E/\mu_{aw} + 1} \quad (7a)$$

$$= \int_0^{U/u^*} \frac{q_{mo}}{q_{mo} + q_t}\,du^+. \quad (7b)$$

For purposes of comparing heat-transfer rates in viscoelastic and purely viscous fluids, and for understanding the differences between these two kinds of material behavior under

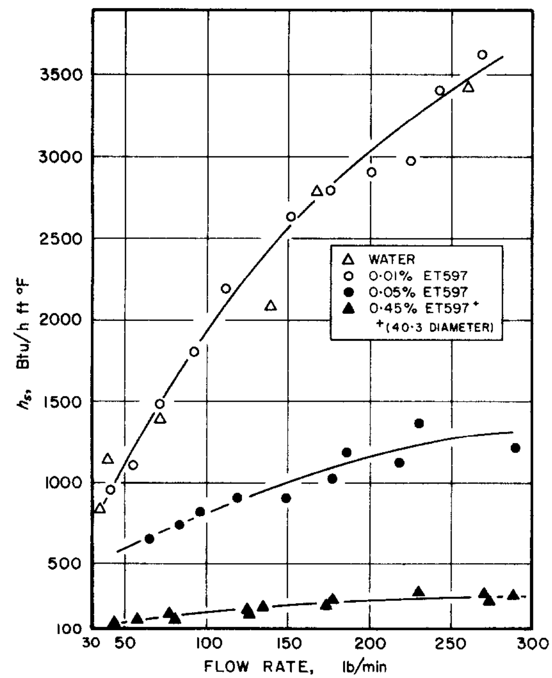

FIG. 6. Measured local heat-transfer coefficient—flowrate relationships as a function of concentration. Values for water and the two dilute solutions are the well-developed coefficients; values for 0·45% solution are not well-developed and represent data taken at $x/D = 40{\cdot}3$.

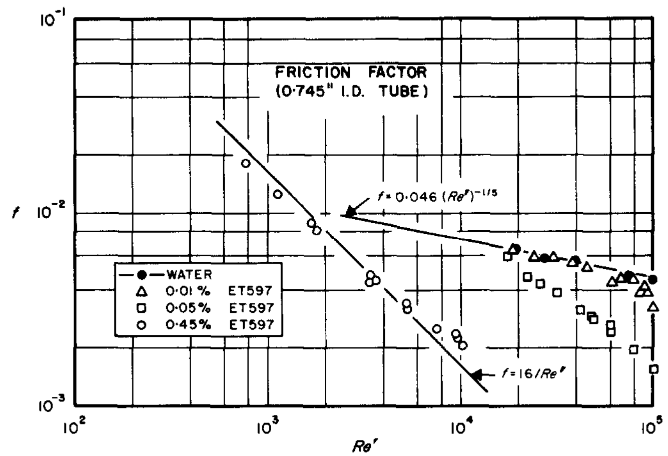

FIG. 7. Friction factor—Reynolds number relationships for fluids used.

Table 2. Comparison of reductions in drag and heat-transfer rates

Flow rate (lb/min)	Reduction in drag ET597 (%)			Reduction in heat-transfer rate ET597 (%)		
	0·01	0·05	0·45	0·01	0·05	0·45
100	none	22	−11 (increase)	none	58	89
200	none	44	36	none	62	90

turbulent flow conditions, the heat-transfer rate differences may logically be considered in terms of the four major terms of equation (6a): $f/2$, θ_m, ϕ_m and I. Each will be considered in turn.

Drag coefficient, $f/2$. Lower values of the drag coefficient for viscoelastic fluids imply lower heat-transfer coefficients as compared to purely viscous materials, but as the two terms in the denominator of equation (6a) are of comparable magnitudes under the conditions used the decrease in the heat-transfer rate should be somewhat less than the decrease in drag, if this were the only operative factor. Reference to Figs. 5 and 6 and to Table 2 shows this is obviously not the case here.

Ratio of mean-to-maximum temperature difference, θ_m. This term is taken as unity or very near unity for purely viscous systems [23, 28] since turbulent temperature profiles are very flat in the case of moderate-to-high Prandtl numbers [20]. If the velocity profile is greatly steepened, in this viscoelastic case, this may no longer be true but it should be noted that θ_m can only decrease to values below unity. Decreasing this term would serve to increase the predicted Stanton numbers for viscoelastic fluids [equation (6a)], not to decrease them as required to predict the low values observed experimentally. Thus changes in this term also cannot accommodate the results obtained for viscoelastic fluids.

Ratio of maximum-to-mean velocity, $1/\phi_m$. If the rates of radial momentum transport are lower in viscoelastic fluids as suggested by the lower drag coefficients, then this velocity ratio must be larger than in turbulent purely viscous systems, and in an extreme case could reach a value as high as 2·0 (the laminar value for Newtonian fluids) for viscoelastic systems having a flow behavior index of unity. The minimum possible value is presumably that for purely viscous systems, for which values in the range of 1·18–1·20 are obtained [12, 23].

Table 3 lists the results of calculations for the two fully turbulent fluids in which the ratio $1/\phi_m$ is varied systematically. For each fluid, calculations were made for the lowest and highest Reynolds numbers studied, as well as for one intermediate value. In making these calculations the value of the temperature ratio θ_m was taken as unity and the experimental drag coefficient was employed. The value of the integral I in equations (7a) or (7b) which is required to predict the experimentally observed heat-transfer rates, for each of these chosen values of the velocity ratio $1/\phi_m$, is compared with the value for purely viscous fluids as given by the equation [12, 23]:

$$I_{pv} = 11{\cdot}8\,(Pr_w)^{-\frac{1}{4}}. \tag{8}$$

Table 3 shows that for the 0·05% polymeric solution, in which appreciable viscoelastic effects are noted (Figs. 5–7) no possible variation in the term ϕ_m^{-1} enables the value of the integral to be reduced to the level of that given by equation (8). Conversely, for the 0·01% solution (which exhibited drag coefficients essentially identical to the purely viscous values, Fig. 7) the purely viscous value $\phi_m^{-1} = 1{\cdot}18$, serves adequately.†

The above considerations point to the fact that the only way in which equation (6a) may be used to interpret the heat-transfer results obtained using the significantly viscoelastic 0·05% solution is by changes in the integral term, equation (7a) or (7b).

The integral I. Appreciable increases in the value of the integral imply a much smaller value of the turbulent diffusivity E, i.e. of the turbulent

† This comment deserves clarification, since it is not obvious superficially that numbers as divergent as 4·30 and 6·10 represent good agreement. Noting the form of equation (6), however, one sees that when the two terms in the denominator are of comparable magnitude the technique of lumping all discrepancies into one term doubles the change required in any adjustable parameter. Thus the discrepancies noted correspond to about 20 per cent errors in a predicted heat-transfer coefficient. Neglect of a Sieder–Tate type of viscosity ratio in equation (6a) to account for radial property variations accounts for errors of as much as half this magnitude and in the same direction.

Table 3. Calculations of effects of the velocity ratio $1/\phi_m$ and of the integral of equation (7)

Assumed $1/\phi_m$	Fluid % ET597	Re'	Pr_w	I experimental	I Purely viscous
1·18	0·01	18500	7·23	4·30	6·10
1·18	0·01	60600	8·06	4·03	5·90
1·18	0·01	91800	8·57	5·01	5·77
1·50	0·01	18500	7·23	3·39	6·10
1·50	0·01	60600	8·06	3·06	5·90
1·50	0·01	91800	8·57	4·05	5·77
2·0	0·01	18500	7·23	1·97	6·10
2·0	0·01	60600	8·06	1·56	5·90
2·0	0·01	91800	8·57	2·55	5·77
1·18	0·05	17500	11·38	8·25	5·24
1·18	0·05	77600	10·89	11·06	5·31
1·18	0·05	100500	10·87	9·95	5·32
1·50	0·05	17500	11·38	7·68	5·24
1·50	0·05	77600	10·89	10·07	5·31
1·50	0·05	100500	10·87	8·75	5·32
2·0	0·05	17500	11·38	6·8	5·24
2·0	0·05	77600	10·89	8·43	5·31
2·0	0·05	100500	10·87	6·93	5·32

transport term q_t, as compared to purely viscous systems. A number of recent analyses and experimental studies of turbulence in viscoelastic fluids have suggested that the principal effects of viscoelastic properties are likely to be confined to the high wavenumber region of the turbulent spectrum [5, 10, 32, 33, 34, 37], though no firm confirmation of these predictions and tentative observations is in fact available. Thus, if a conventional turbulent core exists, in which the radial transport rates are determined primarily by the low wavenumber portion of the spectrum [17], little difference between the heat-transfer rates in viscoelastic and inelastic fluids would be expected in this portion of the velocity field. However, at the Prandtl number levels of interest (Table 3) the principal resistance to heat transfer is likely to be found close to the tube wall. Should this coincide with the region in which generation of turbulence due to small eddies predominates [17], then any "cut-off" of the spectrum for viscoelastic fluids at high wavenumbers [34] could have a profound influence on turbulent transport rates in this region and consequently in the entire tube. Assuming the cut-off to occur when the Deborah number† (Deb = $\phi_{fl}N$) reaches values of the order of unity, and calculating fluid relaxation times from the property measurements reported by Oliver [26], one predicts the cut-off in the turbulent spectrum to occur at wavenumbers of the order of 10^2 cm^{-1}. This would appear to fall into the region of dominant energy levels only close to the tube wall [17] but, as this is the very region in which most of the resistance to heat transfer occurs, it could cause strong changes in the temperature field and lead to large decreases in the heat-transfer rates, as observed. Whether there is such a direct relationship between the fluid relaxation time and the turbulent structure, however, is a question which cannot yet be answered with certainty. Exploration of the temperature fields in these systems, possibly employing higher fluid temperatures to reduce the Prandtl number in order to obtain measurable effects further from the tube wall, appears to present an opportunity for obtaining at least indirect information on the turbulent spectrum in these systems. Noting that the small heated probes normally used for spectral measurements in aerodynamic studies are at least sometimes inoperative in viscoelastic fluids [1, 22] such indirect measurements may in fact be of rather great interest in this case.

† The Deborah number is the ratio of the characteristic time of the fluid to the characteristic time of the process and is discussed in some detail in recent publications [3, 24, 25].

Thermal entry length characteristics

Data as shown in Fig. 4 were used to obtain thermal entry length magnitudes. For the 0·01% fluid these were found to be the same as for turbulent Newtonian fluids [2, 15, 16], namely from ten to twenty-five diameters over the generalized Reynolds number range of 18000–92000. For the 0·05% fluid the entrance length values exceed the Newtonian values and range from ten to thirty diameters for generalized Reynolds numbers of 18000–100000. This behavior is consistent with the steady state heat-transfer performance which suggested decreased values of the eddy transport of heat with increased concentration. In the case of the 0·45% fluid the entry lengths were all greater than the forty-eight diameter length of the test section and the heat-transfer coefficients at the downstream end of the test section were all within 25 per cent of the values calculated for laminar flow of non-Newtonian fluids, even at the highest Reynolds numbers studied [15]. These observations are consistent with one another as the predicted thermal entry lengths for the constant heat flux laminar flow case [29] are all greater than the test section length employed. The observation that the flow was essentially laminar under all conditions is of interest in connection with Fig. 7: low values of the drag coefficient could conceivably be due to either a turbulent but non-dissipative velocity field or to flow conditions which are essentially laminar. The present results suggest that the latter to be the case.

Concluding remarks

It has been shown elsewhere [1, 6, 22, 31] that conventional probes usually employed in turbulence studies—both impact probes for mean (time-averaged) velocity determinations and heated probes used to obtain fluctuating velocity components—may be inoperative or lead to erroneous inferences when used in viscoelastic fluids. Thus, measurements of the heat-transfer and of the time-averaged temperature profiles, rather than being a mere adjunct to other experimental techniques, may represent primary though indirect sources of information concerning the turbulent spectrum in these materials. For example, it was not clear from the previously available fluid mechanical studies whether drag reduction under turbulent flow conditions arises as a result of a strong suppression of all turbulence in the fluid or whether, on the other hand, turbulent velocity fluctuations are present much as usual but the predominance of elastic properties at high wavenumbers renders the turbulent field less dissipative. The present results clearly show that the first of these mechanisms is the dominant one, at least in the region close to the tube wall in which the major part of the heat-transfer resistance may be found under the range of Prandtl numbers employed. Further studies aimed at developing the point further would appear to be cogent.

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REFERENCES

1. A. J. Acosta and D. F. James, Private communication (1966).
2. R. W. Allen, Measurements of friction and local heat-transfer for turbulent flow of a variable property fluid (water) in a uniformly heated tube, Ph.D. Thesis, University of Minnesota, Minneapolis, Minn. (1959).
3. G. Astarita, Two dimensionless groups relevant in analysis of steady flows of viscoelastic materials, *Ind. Engng Chem. Fundls* **6**, 257 (1967).
4. G. Astarita and G. Marrucci, Heat transfer in viscoelastic liquids in turbulent pipe flow, in *Proceedings of the 36th International Congress on Industrial Chemistry*, Brussels (1966).
5. G. Astarita, Possible interpretation of the mechanism of drag reduction in viscoelastic liquids, *Ind. Engng Chem. Fundls* **4**, 354 (1965).
6. G. Astarita and L. Nicodemo, Velocity distributions and normal stresses in viscoelastic turbulent pipe flow, *A.I.Ch.E. Jl* **12**, 478 (1966).
7. S. E. Charm, Calculation of center-line temperatures in tubular heat exchangers for pseudoplastic fluids in streamline flow, *Ind. Engng Chem. Fundls* **1**, 79 (1962).
8. E. B. Christiansen and S. E. Craig, Jr., Heat transfer to pseudoplastic fluids in laminar flow, *A.I.Ch.E. Jl* **8**, 154 (1962).
9. D. W. Dodge and A. B. Metzner, Turbulent flow of non-Newtonian systems, *A.I.Ch.E. Jl* **5**, 189 (1959); **8**, 143 (1962).
10. C. Elata and M. Poreh, Momentum transfer in turbulent shear flow of an elastico-viscous fluid, *Rheol. Acta* **5**, 148 (1966).
11. I. Etter and W. R. Schowalter, Unsteady flow of an Oldroyd fluid in a circular tube, *Trans. Soc. Rheol.* **9** (2), 351 (1965).
12. W. L. Friend and A. B. Metzner, Turbulent heat transfer inside tubes and the analogy among heat, mass and momentum transfer, *A.I.Ch.E. Jl* **4**, 393 (1958).
13. R. F. Ginn and A. B. Metzner, Normal stresses in polymeric solutions, in *Proceedings of the 4th International Congress on Rheology*, p. 583. Interscience, New York (1965).
14. D. F. Gluck, The effect of turbulence promotion on Newtonian and non-Newtonian heat-transfer rates, M.Ch.E. Thesis, Univ. of Delaware, Newark, Del. (1959).
15. M. K. Gupta, Turbulent heat transfer characteristics of viscoelastic fluids, M.M.E. Thesis, University of Delaware, Newark, Del. (1966).
16. J. P. Hartnett, Experimental determination of the thermal-entrance length for the flow of water and oil in circular pipes, *Trans. Am. Soc. Mech. Engrs* **77**, 1211 (1955).
17. J. O. Hinze, *Turbulence*. McGraw–Hill, New York (1958).
18. N. N. Kapoor, J. W. Kalb, E. A. Brumm and A. G. Fredrickson, Stress relaxing solids: recoil phenomena, *Ind. Engng Chem. Fundls* **4**, 186 (1965).
19. H. Markovitz, Normal stress measurements on polymer solutions, in *Proceedings of the 4th International Congress on Rheology*, p. 189. Interscience, New York (1965).
20. W. H. McAdams, *Heat Transmission*, 3rd edn. McGraw–Hill, New York (1954).
21. A. B. Metzner, Heat transfer in non-Newtonian fluids, in *Advances in Heat Transfer*, edited by J. P. Hartnett and T. F. Irvine, Jr., Vol. 2. Academic Press, New York (1965).

22. A. B. METZNER and G. ASTARITA, External flows of viscoelastic materials: fluid property restrictions on the use of velocity-sensitive probes, *A.I.Ch.E. Jl* **13**, 550 (1967).
23. A. B. METZNER and P. S. FRIEND, Heat transfer to turbulent non-Newtonian fluids, *Ind. Engng Chem.* **51**, 879 (1959).
24. A. B. METZNER, J. L. WHITE and M. M. DENN, Constitutive equations for viscoelastic fluids for short deformation periods and for rapidly changing flows: significance of the Deborah number, *A.I.Ch.E. Jl* **12**, 863 (1966).
25. A. B. METZNER, J. L. WHITE and M. M. DUNN, Behavior of viscoelastic materials in short-time processes, *Chem. Engng Prog.* **62**(12), 81 (1966).
26. D. R. OLIVER, The expansion–contraction behavior of laminar liquid jets, *Can. J. Chem. Engng* **44**, 100 (1966).
27. D. R. OLIVER and V. G. JENSON, Heat transfer to pseudoplastic fluids in laminar flow in horizontal tubes, *Chem. Engng Sci.* **19**, 115 (1964).
28. A. W. PETERSON and E. B. CHRISTIANSEN, Heat transfer to non-Newtonian fluids in transitional and turbulent flow, *A.I.Ch.E. Jl* **12**, 221 (1966).
29. W. M. ROHSENOW, *Developments in Heat Transfer*, M.I.T. Press, Boston (1964).
30. J. G. SAVINS, A stress-controlled drag reduction phenomenon, *Rheol. Acta.* To be published.
31. J. G. SAVINS, A pilot tube method for measuring the first normal stress difference and its influence on laminar velocity profile determinations, *A.I.Ch.E. Jl* **11**, 673 (1965).
32. J. G. SAVINS, Drag reduction characteristics of solutions of macromolecules in turbulent pipe flow, *J. Soc. Pet. Engrs* **4**, 203 (1964).
33. F. A. SEYER and A. B. METZNER, Turbulent flow properties of viscoelastic fluids, *Can. J. Chem. Engng.* **45**, 121 (1967).
34. F. A. SEYER and A. B. METZNER, Turbulence in viscoelastic fluids, *Proceedings of the 6th Symposium on Naval Hydrodynamics*. Washington (1966).
35. C. R. SHERTZER, The stress state of elastic fluids in viscometric flow, Ph.D. Thesis, University of Delaware, Newark, Del. (1965).
36. C. R. SHERTZER and A. B. METZNER, Measurement of normal stresses in viscoelastic materials at high shear rates, in *Proceedings of the 4th International Congress on Rheology*, p. 603, Interscience, New York (1965).
37. K. SINGH, Non-Newtonian effects on the turbulent energy spectrum function, Ph.D. Thesis, Pennsylvania State University, University Park, Penn. (1966).
38. T. W. SPRIGGS, J. D. HUPPLER and R. B. BIRD, An experimental appraisal of viscoelastic models, *Trans. Soc. Rheol.* **10** (1), 191 (1966).
39. E. A. UEBLER, Pipe entrance flow of elastic liquids, Ph.D. Thesis, University of Delaware, Newark, Del. (1966).
40. R. D. VAUGHN, Heat transfer to non-Newtonian fluids, Ph.D. Thesis, University of Delaware, Newark, Del. (1956).
41. S. VELA, J. W. KALB and A. G. FREDRICKSON, On stress-relaxing solids: part III. Simple harmonic deformation, *A.I.Ch.E. Jl* **11**, 288 (1965).
42. J. L. WHITE and A. B. METZNER, Thermodynamic and heat transport considerations for viscoelastic fluids, *Chem. Engng Sci.* **20**, 1055 (1965).

Résumé—Une analyse des résultats expérimentaux de transport de chaleur dans des gammes très étendues des variables appropriées est présentée afin d'établir les caractéristiques principales de transport de chaleur de ces systèmes et de délimiter les domaines dans lesquels des études plus avancées et plus détaillées peuvent être utiles.

Ces résultats, analysés en fonction des équations d'analogie, suggèrent que les coefficients de transport turbulents diminuent de façon importante, au moins près des parois, lorsque le niveau d'élasticité du fluide mesuré par son temps de relaxation, est augmenté jusqu'à 10^{-3} à 10^{-2} secondes. Cette conclusion provisoire est importante du point de vue de la définition partielle de la structure de la turbulence dans ces systèmes, car les mesures directes avec les types actuels de sondes de turbulence peuvent être difficiles et même impossibles à effectuer. Les résultats actuels suggèrent également que les coefficients de traînée très faibles observés fréquemment avec ces systèmes n'apparaissent pas à cause d'un champ turbulent "conservatif", en opposition à un champ turbulent "dissipatif", mais plutôt à cause d'une suppression importante de la turbulence dans le fluide lorsque le nombre de Deborah de l'écoulement devient suffisamment grand.

Zusammenfassung—Es wird eine Analyse gegeben von experimentellen Ergebnissen des Wärmeübergangs für einen weiten Bereich von Variablen zu dem Zweck, die Hauptcharakteristika dieser Systeme darzustellen und die Bereiche aufzuzeigen, in welchen genauere Untersuchungen nützlich sein können. Bei einer Analyse entsprechend den Analogiegleichungen deuten diese Ergebnisse einen grösseren Abfall in den Wirbeltransportkoeffizienten an, zumindest im Wandbereich, wo die Elastizität der Flüssigkeit, gemessen als Relaxationszeit auf ein Niveau von $10^{-}3$–10^{-2} Sekunden zunimmt. Diese vorläufigen Schlüsse sind wichtig, um die Turbulenzstruktur in diesen Systemen teilweise zu definieren; direkte Messungen mit den üblichen Turbulenzfühlern sind schwierig oder ganz unmögluch. Die gegenwärtigen Ergebnisse lassen auch den Schluss zu, dass die sehr kleinen Widerstandskoeffizienten, die in diesen Systemen häufig beobachtet werden, nicht als Ergebnis von "konservativen" im Gegensatz zu "dissipativen" Turbulenzfeldern entstehen, sondern aufgrund der starken Unterdrückung der Turbulenz in der Flüssigkeit bei hinreichend grosser Deborah-Zahl.

Аннотация—Дается анализ экспериментальных результатов по теполобмену в достаточно широких диапазонах изменения влияющих параметров с целью установления основных теплообменных характеристик этих систем и определения области, где могут быть полезны дальнейшие, более детальные исследования.

Использование уравнений аналогии показывает, что основное снижение коэффициентов турбулентного обмена наблюдается в пристеночных областях, где эластичность жидкости, характеризуемая временем релаксации, возрастает от 10^{-3} до 10^{-2} сек. Этот предварительный вывод имеет значение для оценок структуры турбулентности в этих системах, поскольку непосредственные измерения с помощью обычных турбулентных датчиков выполнить трудно или даже невозможно.

Следовательно, довольно низкие коэффициенты сопротивления трения, обычно наблюдаемые в этих системах, не обусловлены «консервативностью» (в противоположность «диссипативности») турбулентного плоя, а имеют причиной сильное уменьшение турбулентности в жидкости, когда число Дебора в потоке становится достаточно большим.